Chapter 13 Vectors

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1. In this question all distances are in km.

   A ship *P* sails from a point *A*, which has position vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, with a speed of 52 kmh^{-1} in the direction of $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$.

   (a) Find the velocity vector of the ship.

   [1]

   (b) Write down the position vector of *P* at a time *t* hours after leaving *A*.

   [1]

   At the same time that ship *P* sails from *A*, a ship *Q* sails from a point *B*, which has position vector $\begin{pmatrix} 12 \\ 8 \end{pmatrix}$, with velocity vector $\begin{pmatrix} -25 \\ 45 \end{pmatrix}$

   (c) Write down the position vector of *Q* at a time *t* hours after leaving *B*.

   [1]

   (d) Using your answers to **parts (b)** and **(c)**, find the displacement vector $\overline{PQ}$ at time *t* hours.

   [1]

(e) Hence show that $PQ = \sqrt{34t^2 - 168t + 208}$.

[2]

(f) Find the value of *t* when *P* and *Q* are first 2 km apart.

[2]

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2. The position vectors of three points, *A*, *B* and *C*, relative to an origin *O*, are $\begin{pmatrix} -5 \\ -7 \end{pmatrix}$, $\begin{pmatrix} 10 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$ respectively. Given that $\overrightarrow{AC} = 4\overrightarrow{BC}$, find the unit vector in the direction of $\overrightarrow{OC}$.

[5]

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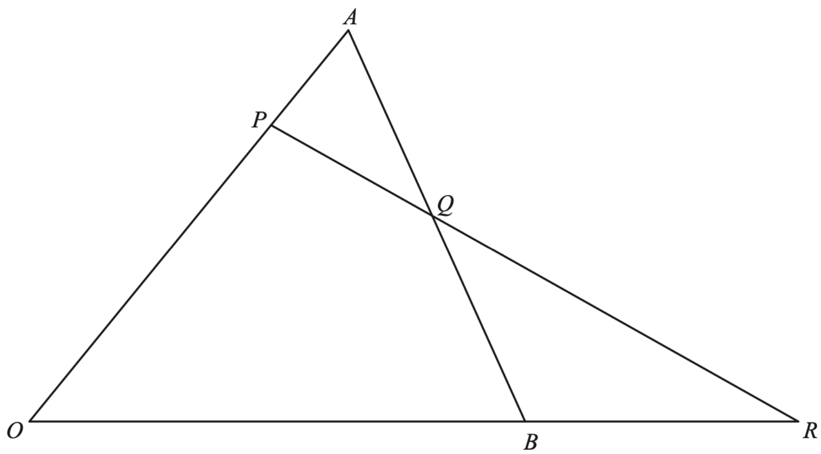
3.



The diagram shows a triangle *OAB* such that $\vec{OA}$ = **a** and $\vec{OB}$ = **b**. The point *P* lies on *OA* such that $\vec{OP} = \frac{3}{4}\vec{OA}$. The point *Q* is the mid-point of *AB*. The lines *OB* and *PQ* are extended to meet at the point *R*. Find, in terms of **a** and **b**,

a. $\vec{AB}$

[1]

b. $\vec{PQ}$, Give your answer in its simplest form.

[3]

It is given that $n\vec{PQ} = \vec{QR}$ and $\vec{BR}$ = k**b**, where n and k are positive constants.

c. Find $\vec{QR}$ in terms of n, **a** and **b**.

[1]

d. Find $\vec{QR}$ in terms of k, **a** and **b**.

[2]

e. Hence find the value of n and of k.

[3]

4. (a) Find the unit vector in the direction of $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$.

[1]

(b) Given that $\begin{pmatrix} 4 \\ 1 \end{pmatrix} + k\begin{pmatrix} -2 \\ 3 \end{pmatrix} = r\begin{pmatrix} -10 \\ 5 \end{pmatrix}$, find the value of each of the constants *k* and *r*.

[3]

(c) Relative to an origin *O*, the points *A*, *B* and *C* have position vectors **p**, 3**q**-**p** and 9**q**-5**p** respectively.

(i) Find $\overrightarrow{AB}$ in terms of **p** and **q**.

[1]

(ii) Find $\overrightarrow{AC}$ in terms of **p** and **q**

[1]

(iii) Explain why *A*, *B* and *C* all lie in a straight line.

[1]

(iv) Find the ratio *AB* : *BC*.

[1]

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5. The vectors **a** and **b** are such that **a** = a**i**+**j** and **b** = 12**i**+b**j**.

(a) Find the value of each of the constants a and b such that 4**a**-**b** = (α +3)**i**-2**j**.

[3]

(b) Hence find the unit vector in the direction of **b** - 4**a**.

[2]

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6. A particle P is initially at the point with position vector $\begin{pmatrix}30\\10\end{pmatrix}$ and moves with a constant speed of 10 ms^{-1} in the same direction as $\begin{pmatrix}-4\\3\end{pmatrix}$.

   a. Find the position vector of P after t s.

[3]

As P starts moving, a particle Q starts to move such that its position vector after t s is given by $\begin{pmatrix}-80\\90\end{pmatrix}+t\begin{pmatrix}5\\12\end{pmatrix}$.

   b. Write down the speed of Q.

[1]

   c. Find the exact distance between P and Q when t = 10, giving your answer in its simplest surd form.

[3]

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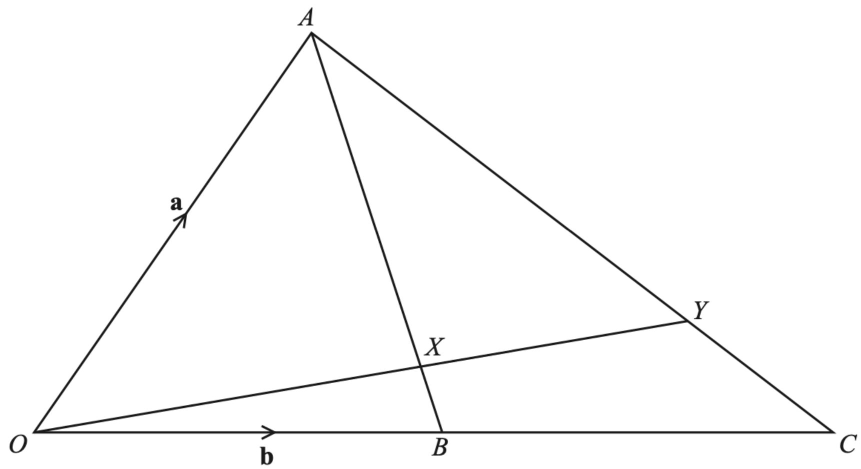
7.



The diagram shows the triangle *OAC*. The point *B* is the midpoint of *OC*. The point *Y* lies on *AC* such that *OY* intersects *AB* at the point *X* where *AX* : *XB* = 3:1. It is given that $\vec{OA}$ = **a** and $\vec{OB}$ = **b**.

(a) Find $\vec{OX}$ in terms of **a** and **b**, giving your answer in its simplest form.

[3]

(b) Find $\vec{AC}$ in terms of **a** and **b**.

[1]

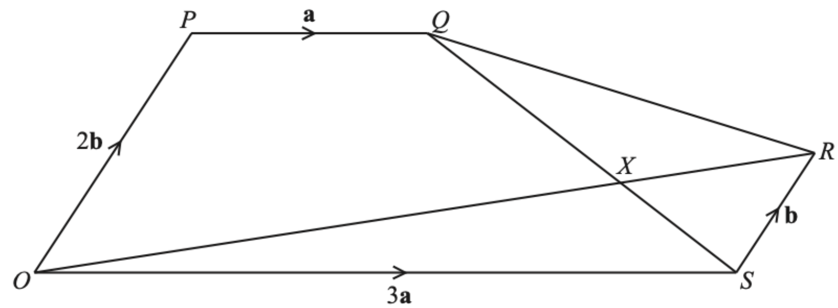
(c) Given that $OY = hOX$, find AY in terms of **a**, **b** and h.

[1]

(d) Given that $AY = mAC$, find the value of h and of m.

[4]

8.



In the diagram $\vec{OP} = 2\mathbf{b}$, $\vec{OS} = 3\mathbf{a}$, $\vec{SR} = \mathbf{b}$ and $\vec{PQ} = \mathbf{a}$. The lines *OR* and *QS* intersect at *X*.

(a) Find $\vec{OQ}$ in terms of **a** and **b**.

[1]

(b) Find $\vec{QS}$ in terms of **a** and **b**.

[1]

(c) Given that $\vec{QX} = \mu\vec{QS}$, find $\vec{OX}$ in terms of **a**, **b** and μ.

[1]

(d) Given that $\vec{OX} = \lambda\vec{OR}$, find $\vec{OX}$ in terms of **a**, **b** and λ.

[1]

(e) Find the value of μ and λ.

[3]

(f) Find the value of $\frac{QX}{XS}$.

[1]

(g) Find the value of $\frac{OR}{OX}$.

[1]